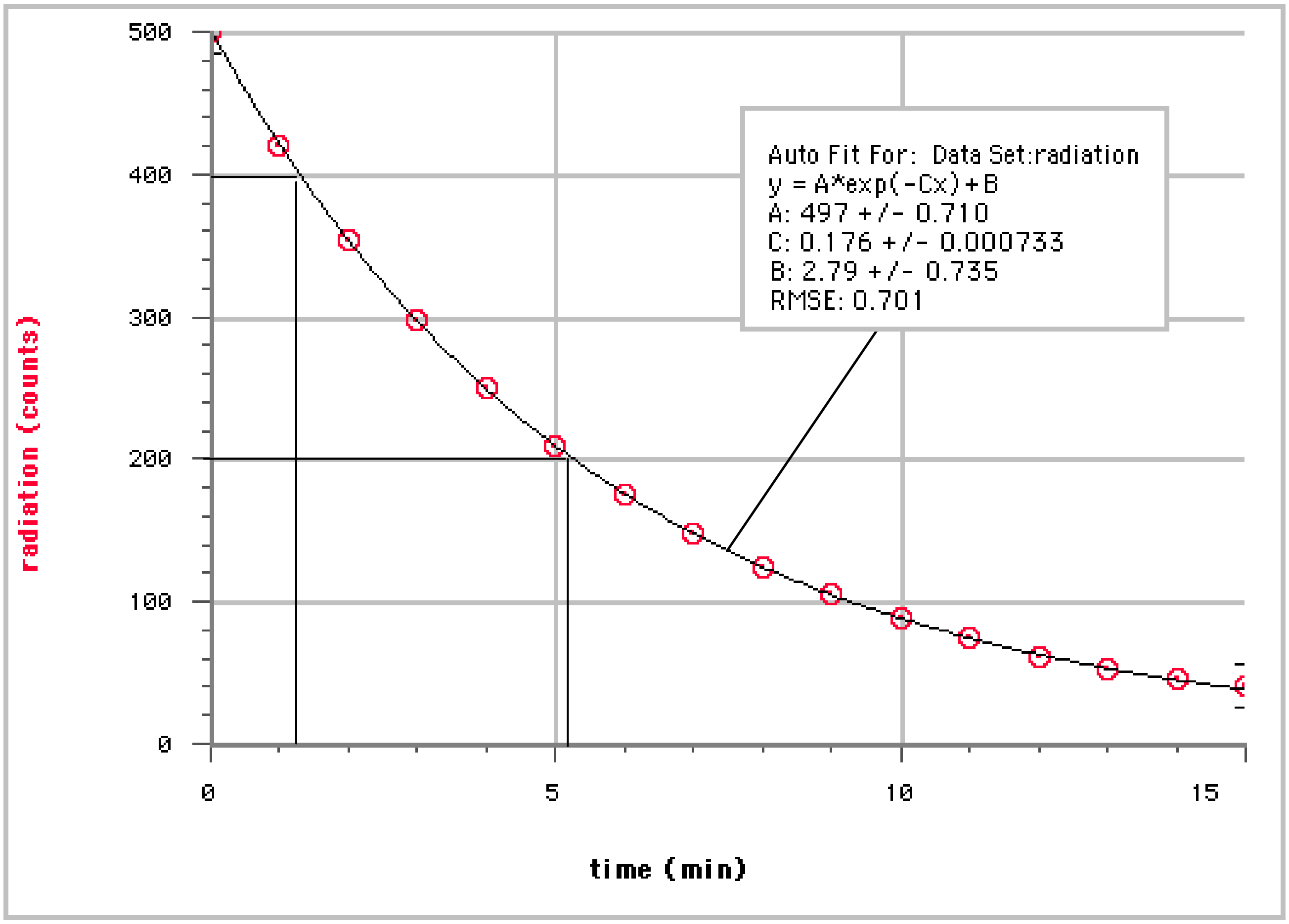
Chemistry – Chapter 10 Notes on Half-Life

Determining 1/2-life

When nuclei undergo radioactive decay, they do so spontaneously. Nothing we can do will quicken or slow the process. The rate at which these nuclei decay depends on how many atoms are in the sample at a given moment. As the size of the sample decreases, the rate of decay slows down in a predictable way. We define the half-life of an isotope as the time required for ½ of the atoms in a sample to undergo decay.



Note that on the graph above, the count rate is 400 at 1.3 min and is 200 at 5.2 min. Whatever the size of the original sample, half of the atoms have decayed in 3.9 min. If you were to find the time for the count rate to drop to 100, you would obtain 9.1 min, another 3.9 min interval. This graphical determination is in close agreement with the algebraic determination using $t_{1/2} = \frac{\ln 2}{C}$. When you perform the calculation, you obtain $t_{1/2}$ = 3.94 min.

Solving ½-life problems

The general form of the equation we use in first-year chemistry is $\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$, where A is the amount remaining, A_0 is the original amount, and n is the number of half-lives. In this course n will be an integer that depends on the total time and the time for one half-life, $n = \frac{T}{t_{1/2}}$.

The key to solving any of the ½-life problems you are likely to face in first year chemistry is to remember that the problem has two parts. *Either* you use information about the fraction remaining to find n, and then solve for T or $t_{1/2}$, *or* you use information about the time to determine n, then solve for either the amount remaining or the original amount. In both cases, *your first step is to find n.*

Example 1

The half-life of $_{95}Am^{241}$ is 458 years. How much of a 12.0 g sample would remain after 1375 years?

Since you know both the total time and the half-life, use these to find n.

$n = \frac{1375}{458} \approx 3$ Approximately 3 half-lives have gone by in 1375 years. So the fraction remaining, $\frac{A}{A_0}$, equals $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$. Thus, the amount remaining is $1/8$ of 12.0 g or 1.50 g.

Example 2

Cobalt-60 is commonly used as a source of beta particles. Its half-life is 5.26 years. How long would it take for 93.75% of a sample of Co-60 to decay?

The fraction of Co-60 remaining must be 100 – 93.75% or 6.25% or 0.0625. The common fraction equivalent is $1/16$, which is $\left(\frac{1}{2}\right)^4$. So, 4 half-lives have gone by. The total time must be 4 x 5.26 yr = 21.0 yr.

Challenge

Some of you might ask, "How could I solve the problem if the number of half-lives is not an integer?" This is one of the reasons you learn logarithms. Suppose in example 2, we wanted to know how long it would take for the fraction remaining to be 10% (or 0.10)? The equation is $0.10 = \left(1/2\right)^n$. We know that the answer must be somewhere between 3 half-lives (12.5% or $1/8$) and 4 half-lives (6.25% or $1/16$). To solve for the exponent, take the natural log of both sides $\ln(0.10) = n \times \ln\left(1/2\right)$.

Rearrange to obtain $n = \frac{\ln(0.10)}{\ln\left(1/2\right)} = 3.32$. The time must be 3.32 x 5.26 = 17.5 yrs.